The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1 ***Solution Set 2.0***

Date: October 5, 2017 Course: EE 313 Evans

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Last, First

- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- ***Please disable all wireless connections on your calculator(s) and computer system(s).***
- Please turn off all cell phones.
- No headphones are allowed.
- All work should be performed on the midterm exam. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

Problem	*Point Value*	*Your score*	*Topic*
1	21		Sampling Sinusoids
2	21		Fourier Series
3	21		Child of Fourier Series
4	18		Spectrograms
5	19		Potpourri
Total	100		

Problem 1.1 *Sampling Sinusoids*. 21 points.

Consider the sinusoidal signal $x(t) = \cos(2\,\pi\, f_0\, t)$ for continuous-time frequency f_0.

We then sample $x(t)$ at a sampling rate f_s to produce a discrete-time signal $x[n]$.

(a) Derive the formula for $x[n]$. *6 points.*

$$\boldsymbol{x[n] = x(t)|_{t=nT_s} = \cos(2\pi f_0(nT_s)) = \cos(2\pi f_0 T_s n) = \cos\left(2\pi \frac{f_0}{f_s} n\right) = \cos(\hat{\omega}_0 n)}$$

SPFirst Sec. 4-1

(b) Based on your answer in part (a), give a formula for the discrete-time frequency $\hat{\omega}_0$ of $x[n]$ in terms of the continuous-time frequency f_0 and sampling rate f_s. *6 points*.

$$\boldsymbol{\hat{\omega}_0 = 2\pi \frac{f_0}{f_s}}$$

SPFirst Sec. 4-1.1

(c) Plot the spectrum of $x[n]$ over discrete-time frequencies from -3π rad/sample to 3π rad/sample, assuming that the sampling theorem has been satisfied, i.e. $f_s > 2\,f_0$. *9 points.*

Since $f_s > 2\,f_0$, we have $\frac{f_0}{f_s} < \frac{1}{2}$ and hence $0 \le \hat{\omega}_0 < \pi$.

SPFirst Sec. 4-2.2

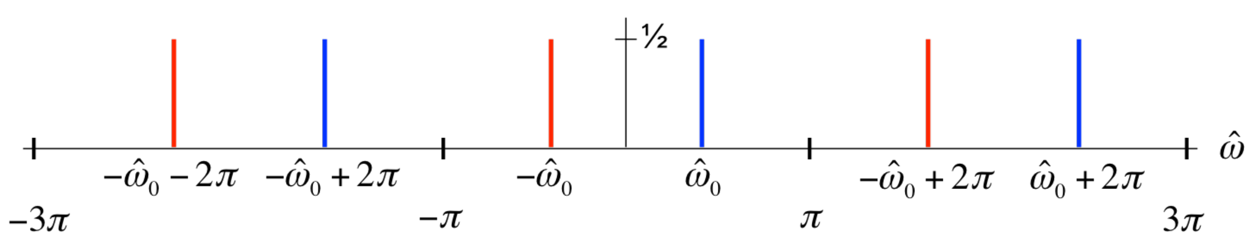
The signal $x[n]$ can be written as $x[n] = \cos(\hat{\omega}_0 n) = \frac{1}{2} e^{-j\hat{\omega}_0 n} + \frac{1}{2} e^{j\hat{\omega}_0 n}$

Hence, $x[n]$ has frequency components at $\hat{\omega}_0$ and $-\hat{\omega}_0$, each with strength ½.

Moreover, the discrete-time frequency domain is periodic with period 2π because

$$\boldsymbol{y[n] = \cos((\hat{\omega}_0 + 2\pi)n) = \cos(\hat{\omega}_0 n + 2\pi n) = \cos(\hat{\omega}_0 n) = x[n]}$$

SPFirst Sec. 4-2.1



SPFirst Sec. 4-2.2

Problem 1.2 *Fourier Series.* 21 points.

An A major chord on the Western scale consists of notes A, C# and E being played at the same time. The signal corresponding to an A major chord would be

$$x(t) = \cos(2\,\pi\, f_A\, t) + \cos(2\,\pi\, f_{C\#}\, t) + \cos(2\,\pi\, f_E\, t)$$

For the note frequencies, please use $f_A = 440$ Hz, $f_{C\#} = 550$ Hz and $f_E = 660$ Hz.

Consider the Fourier series synthesis formula

$$x(t) = \sum_{k=-N}^{N} a_k e^{j2\pi(k f_0)t}$$

SPFirst Sec. 3-6

HW 1.5 & 2.1

(a) What is the fundamental frequency for the three notes? *6 points*.

SPFirst Sec. 3-3 Introduction (p. 43)

f_0 = gcd(440 Hz, 550 Hz, 660 Hz) = 110 Hz

SPFirst Sec. 3-4.1

(b) Give the value for the Fourier series coefficient a_0. *3 points*.

$$\boldsymbol{a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)dt = \textbf{Average}\{\cos(2\,\pi\, f_A\, t) + \cos(2\,\pi\, f_{C\#}\, t) + \cos(2\,\pi\, f_E\, t)\}}$$

$$\boldsymbol{a_0 = \textbf{Average}\{\cos(2\,\pi\, f_A\, t)\} + \textbf{Average}\,\{\cos(2\,\pi\, f_{C\#}\, t)\} + \textbf{Average}\{\cos(2\,\pi\, f_E\, t)\} = 0}$$

For $0 \le t < T_0$, there are 4, 5, and 6 periods, respectively, for the cosines for notes A, C#, and E.

(c) What is the value of N? *3 points*.

SPFirst Sec. 3-3 Introduction (p. 43)

$f_A = 4\, f_0$ and $f_{C\#} = 5\, f_0$ and $f_E = 6\, f_0$ so $N = 6$.

(d) Give the values for all the Fourier series coefficients a_k for $k = -N, \ldots, 0, \ldots, N$. *9 points*.

We can write $x(t) = \cos(2\,\pi\, f_1\, t) = \frac{1}{2}e^{-j2\pi f_1 t} + \frac{1}{2}e^{j2\pi f_1 t}$ using the "inverse" Euler formula.

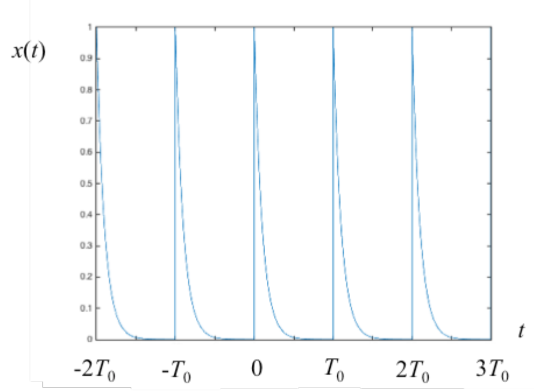
So, $a_{-6} = a_6 = ½$, $a_{-5} = a_5 = ½$, and $a_{-4} = a_4 = ½$.

SPFirst Sec. 3-2.2

All other a_k terms are zero.

Problem 1.3. *Child of Fourier Series.* 21 points**.**

Consider the following periodic signal:



Over one period of time $0 \le t < T_0$, where $T_0 = 10$ seconds,

$$x(t) = e^{-t}$$

Compute the Fourier series coefficients using

SPFirst Sec. 3-4.1

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-j2\pi(kf_0)t}dt$$

HW 2.2 & 3.1

For the value of $x(t)$ at $t = T_0 = 10$ seconds, please use $x(T_0) = e^{-T_0} = e^{-10}$

$$\boldsymbol{a_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-j2\pi(kf_0)t}dt = \frac{1}{T_0}\int_0^{T_0} e^{-t}e^{-j2\pi(kf_0)t}dt = \frac{1}{T_0}\int_0^{T_0} e^{-(1+j2\pi kf_0)t}dt}$$

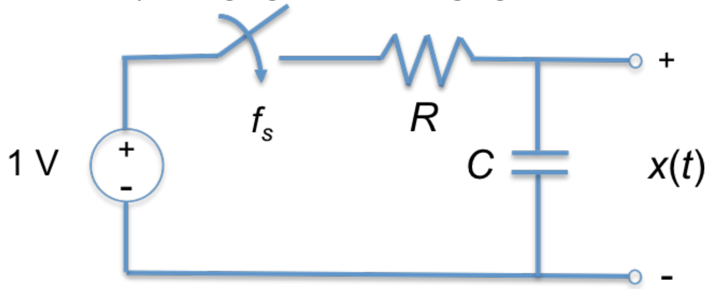
$$\boldsymbol{a_k = -\frac{1}{T_0}\frac{1}{(1+j2\pi kf_0)}e^{-(1+j2\pi kf_0)t}\Big]_0^{T_0} = -\frac{1}{T_0}\frac{1}{(1+j2\pi kf_0)}\left(e^{-(1+j2\pi kf_0)T_0} - 1\right)}$$

$$\boldsymbol{a_k = -\frac{1}{T_0+j2\pi k}\left(e^{-(T_0+j2\pi k)} - 1\right) = \frac{1}{T_0+j2\pi k}\left(1 - e^{-10}e^{-j2\pi k}\right) = \frac{1-e^{-10}}{T_0+j2\pi k}}$$

From the above formula, $\boldsymbol{a_0 = \frac{1}{T_0}(1 - e^{-T_0})}$**.**

Note that $e^{-10} = 4.54 \times 10^{-5}$ **and** $e^{-j2\pi k} = 1$ **.**

Note: An example of where the above period signal *x*(*t*) might occur is in a circuit that is periodically charging and discharging, such as the following circuit where $\tau = RC = 1$:



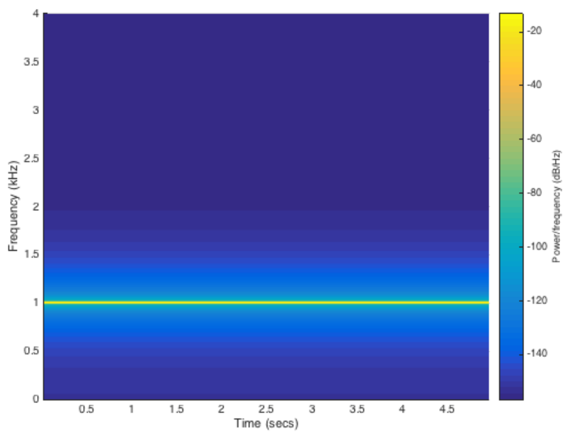
Problem 1.4 *Spectrograms.* 18 points.

For each spectrogram below, give a formula for the signal in either continuous time or discrete time.

Each signal was sampled at a sampling rate of f_s = 8 kHz for 5 seconds.

Each spectrogram used a 1024-point window and a shift of 512 samples.

(a)

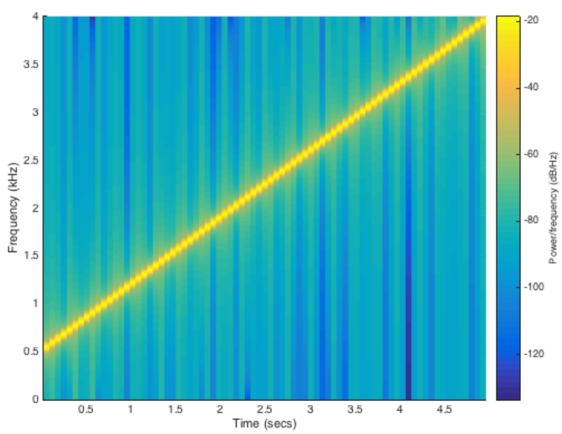


The maximum intensity is a horizontal line at frequency 1 kHz. The frequency resolution on the vertical axis is f_s / 1024 = 7.8125 Hz. The bandwidth of the signal cannot be any more than 7.8125 Hz based on the spectrogram.

$x(t) = \cos(2\pi f_0 t)$ where f_0 = 1000 Hz.

SPFirst Sec. 3-7

(b)



The principal frequency component is increasing over time linearly from 0.5 kHz to 4 kHz over the 5s of time duration. This is a chirp signal.

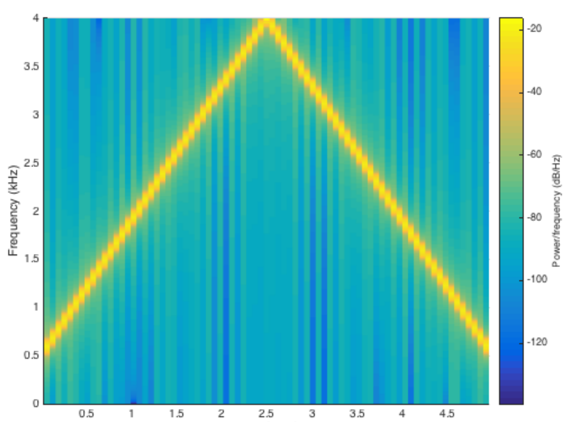
$x(t) = \cos(2\pi\mu t^2 + 2\pi f_0 t)$

where $\mu = (f_2 - f_1) / (2\, t_{\max})$, $f_0 = f_1$ = 500 Hz, f_2 = 4000 Hz, and $t_{\max}$ = 5s

SPFirst Sec. 3-8

HW 2.3 & 3.4

(c)



Note: Phase not available in spectrograms.

The principal frequency component is increasing linearly from 0.5 to 4 kHz over the first 2.5s and then decreasing linearly from 4 to 0.5 kHz over the last 2.5s. With sampling rate of f_s = 8 kHz, signal frequencies above 4 kHz will alias. This is a chirp signal of increasing frequency that experiences folding when the principal frequency exceeds 4 kHz.

$x(t) = \cos(2\pi\mu t^2 + 2\pi f_0 t)$

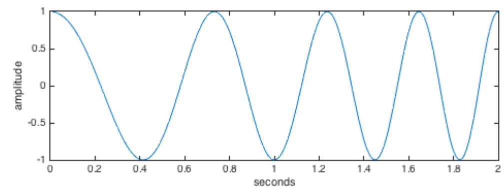
where $\mu = (f_2 - f_1) / (2\, t_{\max})$, $f_0 = f_1$ = 500 Hz, f_2 = 7500 Hz, and $t_{\max}$ = 5s

SPFirst Sec. 3-8

Tune-Up #4 | *HW 2.3 & 3.4*

Problem 1.5. *Potpourri.* 19 points.

(a) An example of a sinusoidal signal whose primary frequency component is varying with time is shown on the right.



Describe a time-domain only method to determine the primary frequency component as it varies over time. *9 points*.

Signal appears to be sinusoidal and its principal frequency increases over time. It's a chirp.

For a sinusoidal signal, the time between two zero crossings is one half of the period.

Method: **At the first zero crossing, record the time of occurrence as t_0. At the nth zero crossing thereafter, record the time of occurrence as time t_n, then compute the difference in time between the two most recent zero crossings $\Delta t = t_n - t_{n-1}$, then compute $T_n = 2\ \Delta t$ and finally compute $f_n = 1 / T_n$.**

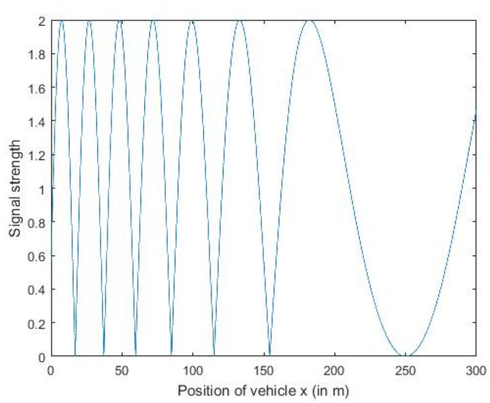
HW 1.1

SPFirst Sec. 2-3 & 3-8

Discussion during "Reconstruction" movies from *SPFirst* Sec. 4-4

(b) While standing outside on campus, you make a voice call on a cell phone. The person you called answers. As you're talking, the voice level slowly fades in and out, but the call does not drop. To get a better voice connection, would it be better to hang up and call again, or is it better to walk while talking? Why? Please give an answer based on the material covered in class. *10 points*.

Mini-Project #1 investigated the effect of multipath fading on the quality of a wireless signal transmission at a receiver. Mini-Project #1 involved a single bounce path. The reflection occurs at the same point in space and has the same effect on the transmitted signal. The transmitter is stationary. Received signal strength is measured at different points in space, and the measurement does not depend on how the receiver got to that point in space:



For the situation described in midterm problem 1.5(b), one is standing outside and receiving a low signal strength transmission. Assume the scenario in Mini-Project #1. If the person hangs up and calls again, then the multipath fading would not have changed. So, it is better to start walking until one finds a high enough received signal strength.

Mini-Project #1 Solution Set

MATLAB Code to Generate Figures Used on the Midterm #1 Exam

```
% Problem 1.3
tau = 1;
T0 = 10*tau;
Ts = tau/1000;
fs = 1 / Ts;
N = 5;
t = -((N-1)/2)*T0 : Ts : ((N+1)/2)*T0;
x = exp(-mod(t,T0)/tau);
plot(t, x);
ylim( [0 1] );

% Problem 1.4
fs = 8000;
Ts = 1 / fs;
tmax = 5;
t = Ts : Ts : tmax;
f0 = 1000;
x = cos(2*pi*f0*t);
spectrogram(x, 1024, 512, 1024, fs, 'yaxis');

% Problem 1.4(b)
fs = 8000;
Ts = 1 / fs;
tmax =  5;
t = 0 : Ts : tmax;
f1 =  500;
f2 = 4000;
f0 =   f1;
mu = (f2-f1)/(2*tmax);
angle = 2*pi*mu*t.^2 + 2*pi*f0*t;
x = cos(angle);
spectrogram(x, 1024, 512, 1024, fs, 'yaxis');

% Problem 1.4(c)
fs = 8000;
Ts = 1 / fs;
tmax =  5;
t = 0 : Ts : tmax;
f1 =  500;
f2 = 7500;
f0 =   f1;
mu = (f2-f1)/(2*tmax);
angle = 2*pi*mu*t.^2 + 2*pi*f0*t;
x = cos(angle);
spectrogram(x, 1024, 512, 1024, fs, 'yaxis');

% Problem 1.5(a)
time = 2;                % signal duration (seconds)
fs = 100;                % sampling rate
Ts = 1/fs;               % sampling time
t = Ts : Ts : time ;     % create a time vector
f0 = 1;                  % initial frequency
fstep = 1;               % frequency slope
phi = pi*fstep*t.^2;     % linearly increase frequency
x=cos(2*pi*f0*t + phi);
plot(t, x);
xlabel('seconds');
ylabel('amplitude');
```